

## AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

### Listing of Claims:

1           1. (Currently amended) A method for bounding the solution set of a  
2   system of linear equations  $\mathbf{Ax} = \mathbf{b}$ , wherein  $\mathbf{A}$  is an interval matrix and  $\mathbf{b}$  is an  
3   interval vector, the method comprising:  
4       receiving the system of linear equations  $\mathbf{Ax} = \mathbf{b}$ ;  
5       storing  $\mathbf{Ax} = \mathbf{b}$  in a memory in a computer system;  
6       preconditioning the set of linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through  
7   by a matrix  $\mathbf{B}$  to produce a preconditioned set of linear equations  $\mathbf{BAx} = \mathbf{Bb}$ ,  
8   wherein the set of linear equations is a representation of a global optimization  
9   problem;  
10       substituting  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ , wherein  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$  to produce  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;  
11       widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
12   midpoints of the interval elements of  $\mathbf{M}$  form the identity matrix; and  
13       using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull  $\mathbf{h}$  of the system  $\mathbf{Mx} = \mathbf{r}$ , which bounds  
14   the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ .

1           2. (Currently amended) The method of claim 1, wherein the method  
2   further comprises computing the matrix  $\mathbf{B}$  by:  
3       computing an approximate center  $\mathbf{A}_C$  of the interval elements of matrix  $\mathbf{A}$ ;  
4   and  
5       forming  $\mathbf{B}$  by computing an approximate inverse of  $\mathbf{A}_C$ ,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .

1           3 (Canceled).

1           4. (Currently amended) The method of claim 1, further comprising  
2     |     assuring that  $\sup(r_i) \geq 0$  by changing the sign of  $r_i$  ~~[[[]]]~~ and  $x_i$  ~~[[[]]]~~ if necessary,  
3     |     wherein  $r_i$  is an element of  $\mathbf{r}$ .

1           5. (Original) The method of claim 1, further comprising:  
2           determining if  $\mathbf{M}$  is regular; and  
3           using the Gauss-Seidel process for computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not  
4     regular.

1           6. (Currently amended) A computer-readable storage medium storing  
2     instructions that when executed by a computer cause the computer to perform a  
3     method for bounding the solution set of a system of linear equations  $\mathbf{Ax} = \mathbf{b}$ ,  
4     wherein  $\mathbf{A}$  is an interval matrix and  $\mathbf{b}$  is an interval vector, the method  
5     comprising:  
6         receiving the system of linear equations  $\mathbf{Ax} = \mathbf{b}$ ;  
7         storing  $\mathbf{Ax} = \mathbf{b}$  in a memory in a computer system;  
8         preconditioning the set of linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through  
9     by a matrix  $\mathbf{B}$  to produce a preconditioned set of linear equations  $\mathbf{BAx} = \mathbf{Bb}$ .  
10    wherein the set of linear equations is a representation of a global optimization  
11    problem;  
12         substituting  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ , wherein  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$  to produce  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;  
13         widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
14    midpoints of the interval elements of  $\mathbf{M}$  form the identity matrix; and  
15         using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull  $\mathbf{h}$  of the system  $\mathbf{Mx} = \mathbf{r}$ , which bounds  
16    the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ .

1           7. (Currently amended) The computer-readable storage medium of claim  
2   6, wherein the method further comprises computing the matrix **B** by:  
3   |           computing an approximate center **A<sub>C</sub>** of the interval elements of matrix A;  
4   and  
5           forming **B** by computing an approximate inverse of **A<sub>C</sub>**,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .

1           8. (Currently amended) The computer-readable storage medium of claim  
2   6, wherein using **M** and **r** to compute the hull **h** involves:  
3           forming **P** as an inverse of the left endpoint of **M**;  
4           forming  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ;  
5           forming  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ , wherein  $e_i^T$  is a unit vector in  
6   | which the  $i$ -th element is 1 and other elements are 0, and wherein  $r_i$  is an element  
7   of **r**;  
8           setting  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ;  
9           setting  $\inf(h_i) = z_i$  if  $z_i \leq 0$ ; and  
10          setting  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ .

1           9. (Currently amended) The computer-readable storage medium of claim  
2   6, wherein the method further comprises assuring that  $\sup(r_i) \geq 0$  by changing the  
3   | sign of  $r_i$  [[ $()$ ] and  $x_i$ [[ $()$ ]] if necessary, wherein  $r_i$  is an element of **r**.

1           10. (Original) The computer-readable storage medium of claim 6, wherein  
2   the method further comprises:  
3           determining if **M** is regular; and  
4           using the Gauss-Seidel process for computing the hull **h** if **M** is not  
5   regular.

1           11. (Currently amended) An apparatus that bounds the solution set of a  
2   system of linear equations  $\mathbf{Ax} = \mathbf{b}$ , wherein  $\mathbf{A}$  is an interval matrix and  $\mathbf{b}$  is an  
3   interval vector, comprising:  
4       a receiving mechanism configured to receive the system of linear  
5   equations  $\mathbf{Ax} = \mathbf{b}$ ;  
6       a storing mechanism configured to store  $\mathbf{Ax} = \mathbf{b}$  in a memory in a  
7   computer system;  
8       a preconditioning mechanism that is configured to precondition the set of  
9   linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through by a matrix  $\mathbf{B}$  to produce a  
10   preconditioned set of linear equations  $\mathbf{BAx} = \mathbf{Bb}$ , wherein the set of linear  
11   equations is a representation of a global optimization problem;  
12       substituting  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ , wherein  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$  to produce  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ ;  
13       a widening mechanism that is configured to widen the matrix  $\mathbf{M}_0$  to  
14   produce a widened matrix  $\mathbf{M}$ , wherein the midpoints of the interval elements of  $\mathbf{M}$   
15   form the identity matrix; and  
16       a hull computing mechanism that is configured to use  $\mathbf{M}$  and  $\mathbf{r}$  to compute  
17   the hull  $\mathbf{h}$  of the system  $\mathbf{Mx} = \mathbf{r}$ , which bounds the solution set of the system  
18    $\mathbf{M}_0\mathbf{x} = \mathbf{r}$ .

1           12. (Currently amended) The apparatus of claim 11, wherein the  
2   preconditioning mechanism is configured to:  
3       compute an approximate center  $\mathbf{A}_C$  of the interval elements of matrix  $\mathbf{A}$ ;  
4       and to  
5       form  $\mathbf{B}$  by computing an approximate inverse of  $\mathbf{A}_C$ ,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .

1           13. (Currently amended) The apparatus of claim 11, wherein the hull  
2   computing mechanism is configured to:  
3       form  $\mathbf{P}$  as an inverse of the left endpoint of  $\mathbf{M}$ ;

4           form  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ;  
5           form  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ , wherein  $e_i^T$  is a unit vector in  
6           which the  $i$ -th element is 1 and other elements are 0, and wherein  $r_i$  is an element  
7           of  $\mathbf{r}$ ;  
8           set  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ;  
9           set  $\inf(h_i) = z_i$  if  $z_i \leq 0$ ; and to  
10          set  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ .

1           14. (Currently amended) The apparatus of claim 11, wherein the  
2           preconditioning mechanism is configured to assure that  $\sup(r_i) \geq 0$  by changing  
3           the sign of  $r_i$  ~~[[()]]~~ and  $x_i$  ~~[[()]]~~ if necessary, wherein  $r_i$  is an element of  $\mathbf{r}$ .

1           15. (Original) The apparatus of claim 11, wherein the preconditioning  
2           mechanism is configured to:  
3           determine if  $\mathbf{M}$  is regular; and to  
4           terminate the process of computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not regular.

1           16. (Currently amended) A method for bounding the solution set of a  
2           system of linear equations  $\mathbf{Ax} = \mathbf{b}$  by multiplying through by the matrix  $\mathbf{B}$  to  
3           produce a preconditioned set of linear equations  $\mathbf{BAx} = \mathbf{Bb}$ , wherein the set of  
4           linear equations is a representation of a global optimization problem, the method  
5           comprising:  
6           receiving the system of linear equations  $\mathbf{Ax} = \mathbf{b}$ ;  
7           storing  $\mathbf{Ax} = \mathbf{b}$  in a memory in a computer system;  
8           substituting  $\mathbf{M}_0 \mathbf{x} = \mathbf{r}$ , wherein  $\mathbf{M}_0 = \mathbf{BA}$  and  $\mathbf{r} = \mathbf{Bb}$  producing  $\mathbf{M}_0 \mathbf{x} = \mathbf{r}$ ;  
9           ~~the method comprising:~~  
10          assuring that  $\sup(r_i) \geq 0$  by changing the sign of  $r_i$  (and  $x_i$ ) if necessary;

11           widening the matrix  $\mathbf{M}_0$  to produce a widened matrix  $\mathbf{M}$ , wherein the  
 12           midpoints of the interval elements of  $\mathbf{M}$  form the identity matrix; and  
 13           using  $\mathbf{M}$  and  $\mathbf{r}$  to compute the hull  $\mathbf{h}$  of the system  $\mathbf{M}\mathbf{x} = \mathbf{r}$ , which bounds  
 14           the solution set of the system  $\mathbf{M}_0\mathbf{x} = \mathbf{r}$  by,  
 15                   forming  $\mathbf{P}$  as an inverse of the left endpoint of  $\mathbf{M}$ ,  
 16                           forming  $c_i = 1/(2P_{ii} - 1)$  for  $i = 1, \dots, n$ ,  
 17                           forming  $z_i = (\inf(r_i) + \sup(r_i))P_{ii} - e_i^T \mathbf{P} \sup(\mathbf{r})$ ,  
 18                           wherein  $e_i^T$  is a unit vector in which the  $i$ -th element is 1  
 19                           and other elements are 0, and wherein  $r_i$  is an element of  $\mathbf{r}$ ,  
 20                           setting  $\inf(h_i) = c_i z_i$  if  $z_i > 0$ ,  
 21                           setting  $\inf(h_i) = z_i$  if  $z_i \leq 0$ , and  
 22                           setting  $\sup(\mathbf{h}) = \mathbf{P} \sup(\mathbf{r})$ .

1           17. (Original) The method of claim 16, further comprising:  
 2           determining if  $\mathbf{M}$  is regular; and  
 3           using the Gauss-Seidel process for computing the hull  $\mathbf{h}$  if  $\mathbf{M}$  is not  
 4           regular.

1           18. (Currently amended) The method of claim 16, wherein the method  
 2           further comprises computing the matrix  $\mathbf{B}$  by:  
 3           |           computing an approximate center  $\mathbf{A}_C$  of the interval elements of matrix  $\mathbf{A}$ ;  
 4           and  
 5           forming  $\mathbf{B}$  by computing an approximate inverse of  $\mathbf{A}_C$ ,  $\mathbf{B} = (\mathbf{A}_C)^{-1}$ .